STABILITY OF FLUID FLOW IN A PLANE CHANNEL WITH UNIFORM INJECTION OR SUCTION THROUGH POROUS WALLS

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The stability of laminar flow in a channel with porous walls is analyzed within the scope of the linear theory.

The stability of flow in the entrance section of a plane channel with one-sided injection through the lower wall has been studied experimentally in several papers [1-3]. The studies show that weak injection lowers the flow stability, while the mechanism of evolution of the disturbances is largely similar to the case of flow on impermeable surfaces and is related to the evolution of Tollmien–Schlichting waves. Under large-injection conditions a flow-stabilizing effect is observed as the injection rate is increased either in the case of accelerated motion in a channel of constant cross section or in nongradient flow realized by regulation of a flexible upper wall. The inception of instability in large injection can be described within the scope of the linear theory of stability of an ideal fluid, and the influence of viscosity is found to be insignificant.

It has been confirmed experimentally [4] that injection in a circular pipe, in contrast with Poiseuille flow, is accompanied by growth of the critical Reynolds number. The transition to turbulent flow is delayed by the stabilizing action of the injection-induced axial pressure gradient (flow acceleration) [4, 5].

The flow stability problem has been analyzed theoretically [6, 7] on the basis of the solution of the modified Orr–Sommerfeld equation for uniform symmetrical injection through the walls in the case of fully developed flow and in the entrance section of a plane channel. The attendant calculations also exhibit the flow stabilization effect and growth of the critical Reynolds number in large injection.

In this work we investigate the influence of flow acceleration on the stability under small disturbances of hydrodynamically fully developed flow in a plane channel with uniform symmetrical injection through the walls, and we demonstrate the feasibility of calculating the neutral curve for large injection in the inviscid approximation. We also discuss the stability of flow in a plane channel with symmetrical suction through the walls.

1. The differential equation for the amplitude of two-dimensional disturbances has the following form on the assumption that the inception of instability is of a local nature [6]:

$$\varphi^{IV} - 2\alpha^2\varphi'' + \alpha^4\varphi = i\alpha\,\mathrm{Re}\,[(F'-c)(\varphi''-\alpha^2\varphi) - F'''\varphi] + \mathrm{R}\,F(\varphi'''-\alpha^2\varphi') - \mathrm{R}\,F''\varphi. \quad (1)$$

The velocity distribution in the hydrodynamically developed flow far from the entrance to the channel is described by the function F(η) [8, 9]:

$$u_x = (U_0 - Vx/h)\,F'(\eta),\quad u_y = VF(\eta).$$

The differential equation for F(η) has the form

$$F^{IV} = \mathrm{R}\,(FF''' - F'F'') \quad (2)$$

with the boundary conditions

$$F(0) = F''(0) = 0,\quad F'(1) = 0,\quad F(1) = 1. \quad (3)$$

The effect of injection or suction on the flow stability is included in Eq. (1) either indirectly [through the dependence of F(η) on the parameter R] or directly in the last two terms. The first of these terms is associated with the presence of a transverse component

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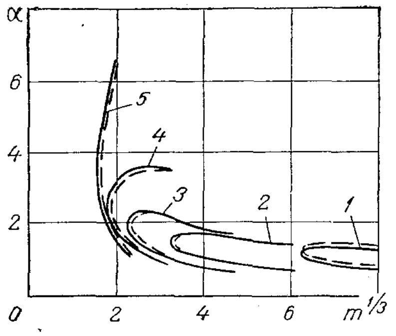

Fig. 1

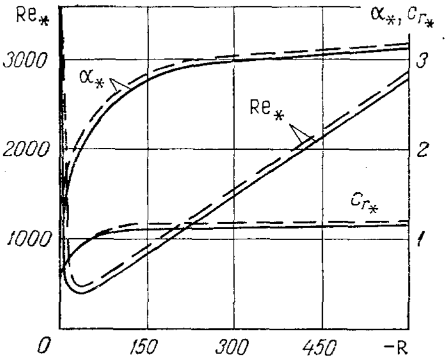

Fig. 2

Fig. 1. Neutral curves. 1) R = −5; 2) −15; 3) −35; 4) − 100; 5) −∞.

Fig. 2. Critical values Re_*, α_*, and c_{r*} versus injection Reynolds number.

of the flow velocity, and the second with flow acceleration (deceleration) induced by injection (suction). For boundary-layer flow, where the second effect is small, or in the case of flow in a plane channel with injection through one wall and suction at the same rate through the other wall, where the investigated effect is absent altogether, the last term in Eq. (1) is also absent [10, 11].

We confine the discussion to symmetrical disturbances, so that the boundary conditions have the form

$$\varphi'(0) = \varphi'''(0) = \varphi(1) = \varphi'(1) = 0. \tag{4}$$

The solutions of the corresponding eigenvalue problems are determined by the method proposed in [12].

2. To determine the role of flow acceleration in the case of injection we calculate the neutral curves (Fig. 1) both from the solution of the complete equation (1) (solid curves) and without the last term in this equation (dashed curves). The variations, corresponding to these neutral curves, of the critical values of the Reynolds numbers Re_*, the wave number α_*, and the phase velocities c_{r*} as a function of the injection parameter $|R|$ are shown in Fig. 2. The results of solving Eq. (1) (solid curves) are in good agreement with the data obtained in [6]. It is evident from the figure that the inclusion of the last term in Eq. (1) does not significantly affect either the shape of the neutral curves or the values of the critical parameters. Contrary to the prevalent notion of the decisive role of flow acceleration in enhancing the flow stability in injection [4, 5], therefore, it is reasonable to infer that the principal factor promoting stabilization of the investigated flow is the presence of a transverse velocity component.

It is also important to note that the failure of Squire's theorem in the given situation is specifically attributable to the presence of an axial velocity gradient. Consequently, the inconsequential effect of flow acceleration on the stability characteristics has the obvious implication that the analysis of flow in a plane channel with injection can be confined to two-dimensional disturbances only, as in the case of constant mass flow rates.

3. In the limit of infinite injection rate $R \to -\infty$ the neutral curves and critical parameters can be calculated without the viscous terms in Eq. (1), i.e., from the solution of the problem

$$i\alpha m\,[(F' - c)(\varphi'' - \alpha^2\varphi)\,F'''\varphi] + F(\varphi''' - \alpha^2\varphi') + F''\varphi' = 0. \tag{5}$$

Here the function $F(\eta)$ is specified from the inviscid limit of Eq. (2):

$$FF''' - F'F'' = 0 \tag{6}$$

and it has the form [9]

$$F = \sin\left(\frac{\pi}{2}\eta\right). \tag{7}$$

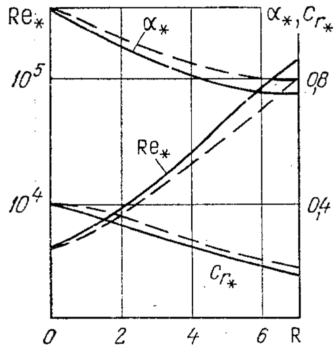

Fig. 3. Critical values Re_*, α_*, and c_{r*} versus suction Reynolds number.

The inviscid neutral curve determined from (5) is the limiting solution of the complete equation (1) as $R \to -\infty$. It is interesting to note that here, unlike the usual situation in the theory of differential equations with a small parameter preceding the leading derivative, the solution (5) is the limit, uniform over the channel cross section, of the solutions of Eq. (1) as $|R| \to \infty$. This result is attributable to the fact that, even though Eq. (5) is an order lower than Eq. (1), its solution still satisfies all the boundary conditions (4), i.e., the condition $\varphi''' = 0$ holds, as is readily verified by differentiating (5) with regard for (7). This result is analogous to one in the theory of large-injection asymptotic solutions of the Navier—Stokes equations, where the inviscid solution is the uniform limit for the complete system of equations as $|R| \to \infty$ [13]. Thus, expression (7), representing the solution of the inviscid third-order equation (6), satisfies all four boundary conditions (3).

Consequently, as in the case of flow in the entrance section of a plane channel [1], the mechanism of the inception of instability in large injection in the fully developed flow zone also has an inviscid behavior. The values of the critical parameters determined from (5) with and without the last term (flow acceleration) are close to one another and equal, respectively: $m_* = 4.46$, $\alpha_* = 3.25$, $c_{r*} = 1.17$ and $m_* = 4.62$, $\alpha_* = 3.31$, $c_{r*} = 1.17$.

4. Figure 3 shows the values of the critical parameters as a function of R, calculated for the suction case on the basis of the solution of Eq. (1) (solid curves) and the conventional Orr—Sommerfeld equation (dashed curves). It is evident from the figure that suction produces considerable flow stabilization, as in the outer flow problem [14]. Moreover, the direct inclusion of the suction effect in (1) causes the calculated critical values to grow more rapidly with the suction rate than when the analysis is based on the conventional Orr—Sommerfeld equation.

NOTATION

x, distance from entrance cross section; y, transverse coordinate measured from axis; u_x, u_y, longitudinal and transverse velocity components of main flow; h, half-width of channel; ν, kinematic viscosity coefficient; U_0, average velocity in entrance cross section; V, suction or injection rate (positive for suction); $U = U_0 - Vx/h$, local average velocity; φ, amplitude of flow disturbances; α, wave number; c, complex phase velocity of disturbances; c_r, real propagation velocity of disturbances; $\eta = y/h$; $Re = Uh/\nu$, Reynolds number of main flow; $R = Vh/\nu$, injection or suction Reynolds number; $m = U/|V|$, injection rate parameter.

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MOLAR MOMENTUM AND HEAT TRANSFER

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Universal relations governing the molar transfer of momentum and heat are derived on the basis of a hypothesis about the dependence of the boundaries of the molar transfer region on the flow structure and with the use of a special mathematical transformation.

Molar transfer, i.e., the transfer of momentum, heat, mass, and other entities by finite masses of a continuum, is commonplace in nature and technology. The well-known molar transfer relations contain empirical constants and are not universal [1].

We now attempt to establish universal relations for steady axisymmetrical and plane molar momentum- and heat-transfer processes in the turbulent core of a turbulent boundary layer.

It has been shown [2] that the following generalized relation holds for molar momentum transfer in a turbulent boundary layer with zero pressure gradient:

$$\frac{d\tilde{U}}{d\tilde{R}} = 1, \tag{1}$$

where $\tilde{U} = (u^+ - 1)/(u^+_\delta - 1)$, $\tilde{R} = \ln y^+/\ln \delta^+$.

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